

## STATISTICAL PHYSICS FOR ANOMALOUS TRANSPORT

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### **1. Introduction**

We have continued the theoretical investigation of the general topic of stable organized motion and coherent structures in plasmas. The main subject for this year was the interaction between the turbulence and the coherent structures and the influence on the energy and particle transport. We have focused on the following four subjects with strong theoretical and experimental relevance.

The work performed under these topics belong to the Subtask “Validation of physics-based transport models” from the Task 5 “First Principle Based Physics Understanding” of “**Physics R&D Needs for the EU Fusion Programme**”. They are also related to the Subtask “ITB scenarios” from Task 3: “Issues for further improvement of ITER operation, or required to prepare later modifications”. Participation at JET through theoretical is planned for 2006 connected to the experiments on transport in H modes (heat, particle and impurities).

The results are presented in 17 publications: 6 articles (two of them in Physical Review Letters), 3 electronic papers and 8 conferences.

#### **1.1. Development of model for the stationary states of the Hasegawa-Mima equation and derivation of the explicit form of the equation, whose solutions are stationary vortices**

All relevant regimes of the ion dynamics are dominated by the polarization drift nonlinearity (the vectorial, or Hasegawa-Mima nonlinearity). We have derived a nonlinear stationary equation, which describes the asymptotic states of the plasma. We have analyzed the solution of this equation for several applications.

#### **1.2. Density pinch in tokamak**

Our original contribution consists of a new physical explanation for the density pinch in tokamak plasma, based on the fundamental process of merging of vortices. We have shown that a constant velocity directed towards the magnetic axis arises from a process of self-organization in which plasma is building up large scale vorticity structures with axisymmetric geometry.

We have also developed an alternative model for density pinch based on test particle approach.

### 1.3. Determination of the diffusion regimes for impurities and fusion products in turbulent plasma at large Larmor radius

The effect of large Larmor radii that characterize ions and impurities on the trapping process due to the ExB drift was studied. Our approach (the decorrelation trajectory method) was extended to the study of the transport induced by the Lorentz force. The transport of impurities and of the fast particles produced by the fusion reaction was analyzed. We have found a new diffusion regime characterized by diffusion coefficient that increases when the Larmor radius increases.

### 1.4. Coherent vortical structures and turbulence in tokamak plasma

The statistical properties of an ensemble of vortices interacting with a turbulent field were determined using the (functional) path integral method. The results consist of a mapping from the spectral local dependence determined in experiment or numerical simulation and the density, amplitude and shape of coherent vortices immersed in drift wave turbulence.

## 2. Results obtained in 2005

### 1.2. Development of model for the stationary states of the Hasegawa-Mima equation and derivation of the explicit form of the equation, whose solutions are stationary vortices

We have investigated the problem of stationary asymptotic states of vortical flows in two-dimensional plasma. In a preceding approach we have derived a compact analytical model for the positive vortices using the analogy of the two-dimensional plasma with the Abelian-Higgs model describing vortices (Abrikosov-Nielesn-Olesen) in superconducting media. A serious extension and reformulation was necessary to be able to describe an arbitrary combination of positive and negative vortices, similar to what it is observed in experiments and numerical simulations in tokamak plasmas.

We have started from the description of the guiding centre plasma by the model of point-like vortices obeying a set of differential equation for the motion of massless objects in plane under a mutual interaction of short spatial range. The original model (Kirchhoff, Onsager, etc.) was devoted to ideal fluid flows (the Euler equation) but there the mutual interaction was long range: the natural logarithm of the relative distance between point vortices. We have generalized this model in a previous work<sup>1</sup> and we have provided for the first time an analytical derivation of the equation *sinh*-Poisson for the asymptotic states of the non-dissipative fluids.

The model of point-like vortices interacting with a short spatial range potential was proposed for neutral fluids and planetary atmosphere by Stewart (1943) and Morikawa (1956) in *meteorology*. The two-dimensional strongly confined plasma (like in tokamak) is described by Hasegawa-Mima equation, which is actually identical to the equation derived by Charney for the planetary atmosphere. The Hasegawa-Mima equation is

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<sup>1</sup> F. Spineanu, M. Vlad, "Self-duality of the relaxation states in fluids and plasmas", **Physical Review E** 67 (2003), 046309

$$(1 - \nabla^2) \frac{\partial \phi}{\partial t} - v_* \frac{\partial \phi}{\partial y} - [-\nabla \phi \times \mathbf{n} \cdot \nabla] \nabla^2 \phi = 0 \quad (1)$$

where  $\phi$  is the velocity stream function (the electric potential),  $\mathbf{n}$  is the direction of the magnetic field and  $v_*$  is the diamagnetic velocity in the poloidal (y)-direction. Although there are numerous numerical simulations, which show the evolution of the turbulent states to ordered structures, an asymptotic equation that describes these structures was not known. We noted above that for 2d ideal fluids that have a similar behavior, the asymptotic structures are solutions of the *sinh*-Poisson.

We have developed a field theoretical approach, which permitted to derive a nonlinear equation for the asymptotic structures of the Hasegawa-Mima equation. The method we have developed for *sinh*-Poisson equation is extended for the short-range interaction of the point vortices. The point-like vortices are represented by a field that is continuous with a kinematic part of the Lagrangean that reflects the minimal coupling to a gauge field where the gauge field is the field that mediates the short-range interaction. The short-range interaction is represented by the Lagrangean density corresponding to the generalization of the helicity, which is the Chern-Simons term. The scalar field, which is connected in this field-theoretical model to the density of point-like vortices, has a self-interaction that reflects the nonlinear vorticity dynamics. The asymptotic states of the Charney-Hasegawa-Mima equation are the self-dual states of the field-theoretical model. We have shown that these states are solutions of the equation

$$\Delta \phi + \frac{1}{2p^2} \sinh \phi (\cosh \phi - p) = 0 \quad (2)$$

where  $p$  is a positive parameter. This is the equation describing the ordered structures appearing at large times in plasma governed by Hasegawa-Mima dynamics. A typical solution that describes a stationary vortex is presented in Figure 1.

These results were published in Physical Review Letters [1].

We have analyzed the solutions of equation (2) and we have shown that they can describe many different physical systems [2], from hurricanes and tornados in atmosphere, to plasma vortices and crystals of plasma vortices [3], [4].

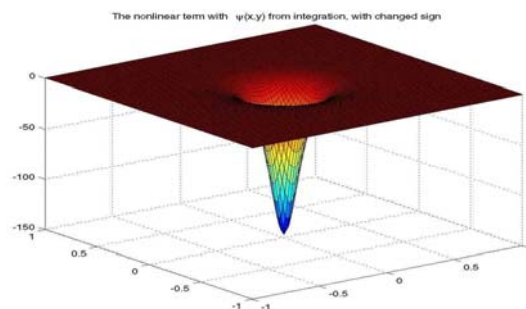


Figure 1. The vorticity obtained from a typical numerical solution of the equation (2), for  $p=1$ . The strong concentration of the vorticity in the center is very clear.

## 2.1. Density pinch in tokamak

There are several attempts to explain the density pinch, all invoking instabilities for which the statistical average of fluctuations of radial displacement and of the density may happen to be directed toward the plasma centre. However this only arises in particular conditions and is not quantitatively sufficient to explain the experimental observations. Without excluding this approaches (thermo diffusion, turbulent equipartition) we have found two more fundamental processes leading to particle pinch.

### 2.1.1. Density pinch in tokamak from the fundamental process of vortices merging

We have shown [5] that there is a fundamental process leading to accumulation of the density with an axisymmetric geometry. This is the vortex merging well known in the case of the ideal fluids. We base our model on the preceding success in the derivation of the equation of the stationary states and the exact solution that we can obtain using the full apparatus of the inverse scattering transform on periodic domain.

We have considered the origin of the vortex merging and creation of ordered states as a fundamental trend to self-organization in two-dimensional plasma. The process is related to the geometrical significance that can be attached to the extremization of a functional defined on the ensemble of states of a string of vorticity in the fluid. We consider a surface constructed from an ensemble of linear vortices parallel to the confining magnetic field and follow it in its time evolution. Since the linear vortices remain in time always perpendicular on the meridional plane, the problem can be reduced to the evolution of a line representing the intersection of the surface with the meridional plane. Now taking this line as an object evolving in time we consider the world-surface that it describes. The world surface is simply the set of successive positions of the line in the meridional plane, but lifted along the time direction. This world surface is in general not a flat one and its geometry can be described by the metric tensor. Using the metric tensor one can calculate the infinitesimal area of an element of the world-surface. In perfect analogy with the motion in space of a particle, the *action functional* for the line can be written as the integral of the infinitesimal area. A set of equations representing the motion of the line in the meridional plane is obtained as the Euler-Lagrange equation at the extremum of the action functional. It has been shown that these equations have the form of the equations for an ideal fluid: equation of continuity for the density and equation of momentum conservation with a specific pressure. It is very interesting that this pressure is *negative* and it has a particular dependence on the density:

$$p = -\lambda\rho^{-1}.$$

This also arises for polytropic gases or Chaplygin gases. The fact that the equations of a perfect fluid with negative pressure results from the simple requirement of the extremum of the element of surface (a geometric property) is fundamental: this must be put in relation with the negative temperature which has been investigated in the past (Taylor, Montgomery, Edwards, etc.) and was associated with the intrinsic tendency of organization of the fluids and plasmas.

We have applied this general analysis to the evolution of a line in the poloidal plane toward the center of the plasma (magnetic axis). Solving the differential equations

for the Chaplygin polytropic gas we found a constant velocity directed towards the magnetic axis. This may explain (or, at least, be one of the contribution to) the density pinch in tokamak. The constant velocity directed towards the magnetic axis is simply arising from a process of self-organization in which plasma is building up large vorticity structures.

We have investigated these self-organized, stationary states, which are attained asymptotically in ideal fluids that are not strongly driven, using a field-theoretical model. This is a formalism developed to represent the properties of a set of point-like vortices interacting by a short-range potential. Analytically the short-range potential is the modified Bessel function of zero order with the space argument normalized at the sonic Larmor radius.

The Eq. (2) is an adequate framework to study the large scale stationary flows in the meridional section. Solving this equation we have obtained a set of flow patterns that can be associated with the equilibrium, in the absence of dissipation and drive. Such a model provides a possible explanation for the flows that exist in tokamak in the equatorial plane leading to accumulation of impurities in the center of the plasma or to the stronger than neoclassical density pinch.

### 2.1.2. Impurity pinch in tokamak from a ratchet-type process

We have also developed an alternative model for density pinch based on test particle approach. This model applies to the evolution of impurities in tokamak plasma and it will be developed in the next period for the study of density. We have shown that an average velocity is produced in turbulent plasmas through a ratchet-type process due to the space variation of the confining magnetic field.

The so-called ratchet process is a generic name for a large class of average stochastic velocities that are generated by unbiased noise. This name suggests the motion of a circular saw with asymmetric saw-teeth. The main ingredients of a minimal model that produces such directed transport consists of a periodic potential (or velocity) with broken reflection symmetry, and a noise (usually a Gaussian white noise). It was shown that, in spite of the asymmetry, the stochastic motion has no systematic preferential direction such that the average velocity is zero at large times. But, if this kind of equilibrium is broken, a ratchet process appears. Thus, a third element has to be included in the minimal model, which can be for instance a driving force, a periodic or stochastic time variation of the amplitude of the periodic potential or of the noise, another noise with different temperature. As shown in a recent review paper<sup>1</sup>, several important results were obtained that permitted a better understanding of these processes with many applications in physics, biology and industry.

We consider an electrostatic turbulence represented by an electrostatic potential  $\varphi(x_1, x_2)$ , where  $\mathbf{x} = (x_1, x_2)$  are the Cartesian coordinates in the plane perpendicular to the confining magnetic field directed along  $z$  axis (slab geometry). The magnetic field depends on the distance from the main symmetry axis as  $\mathbf{B} = B_0 R / (R + x_1)$ , where  $B_0$  is the value of the magnetic field in the origin of the coordinates that is at the distance  $R$  from the symmetry axis. The electrostatic potential is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average. Such a stochastic field is completely determined by the two-point Eulerian correlation function (EC), that is the Fourier

transform of the spectrum (determined from experiments). The test particle motion in the stochastic ExB drift in this potential is described by:

$$\frac{d\bar{x}}{dt} \equiv v(x, t) = -\nabla\phi \times e_z \left(1 + \frac{x_1}{R}\right). \quad (3)$$

We have shown that the inhomogeneity of the confining magnetic field determines a directed transport (an average velocity), although the average drift is zero. The above equation is not similar to a typical model for a ratchet process. However we have shown that it contains the three specific elements mentioned above and discussed in detail in a recent review paper<sup>2</sup>. They are all included in the stochastic drift velocity. The periodic velocity does not appear explicitly in (3) but the solutions for static potentials are periodic functions of time that lie on the contour lines of  $\phi(\mathbf{x})$ . Thus, the Lagrangian velocity  $\mathbf{v}[\mathbf{x}(t)]$  is periodic. The space dependence of the magnetic field produces the symmetry braking of the Lagrangian velocity. The random dynamics is determined by the stochastic potential itself, which plays the role of the Gaussian noise found in the standard model for ratchets. The third element, which drives the system out of equilibrium, is the time variation of the potential. We have show that for static potentials the asymptotic average velocity is zero and that a ratchet process appears when the stochastic potential is time dependent. We have also shown that the ratchet process modeled by equation (1) has the property of current inversion: the sign of the average velocity depends on the parameters of the turbulence.

The average displacement generated by the stochastic equation (3) is determined using the decorrelation trajectory method, a semi-analytical approach we have developed in previous work<sup>3,4</sup>. An average velocity directed along the gradient of the confining magnetic field (along  $x_1$  axis),  $V^R$ , is obtained. It is presented in Figure 2 as function of time. For a static potential (dashed line) the average velocity is positive at small time, then it becomes negative and decays to zero showing that the ratchet process does not appear in this case. The time variation of the stochastic potential determines a finite asymptotic value of the average velocity (solid line).

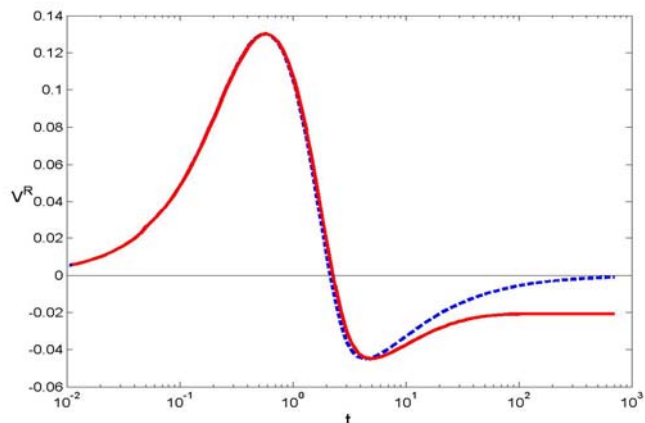


Figure 2: The time dependent average velocity normalized with  $V$  for a static potential (dashed line) and for a time dependent one with  $K=20$  (solid line).

<sup>2</sup> P. Reimann, Phys. Reports **361**, 57 (2002).

<sup>3</sup> M. Vlad, F. Spineanu, J.H. Misguich, R. Balescu, "Diffusion with intrinsic trapping in 2-d incompressible velocity fields", **Physical Review E** **58** (1998) 7359-7368.

<sup>4</sup> M. Vlad, F. Spineanu, "Trajectory structures and transport", **Physical Review E** **70** (2004) 056304(14).

The ratchet velocity obtained for typical tokamak plasma conditions is of the order of 1m/s, which is in agreement with experimental measurements. It is large enough for producing an important effect on the evolution of impurity density. This velocity is in the horizontal plane (on the direction of the gradient of the confining magnetic field). The importance of this original mechanism in the evolution of impurities in **JET** will be analyzed during the experiments scheduled for Campaign 16.

These results were published in [6].

### **2.3. Determination of the diffusion regimes for impurities and fusion products in turbulent plasma at large Larmor radius**

Particle transport determined by a stochastic electric field superposed on a magnetic field was much studied in the guiding center approximation, which corresponds to Larmor radii smaller than the correlation length of the electric field. This approximation is not adequate for ions in the high temperature fusion plasmas. The ratio of the Larmor radius  $\rho$  over the correlation length of the turbulence  $\lambda$  is of the order one for ions and of the order of several tens for the fast particles produced in the fusion reaction. We have considered particles with large Larmor radii and have studied the transport produced by the Lorentz force (*Lorentz transport*). The diffusion coefficient is determined for very large ranges of the parameters in order to identify the possible diffusion regimes that can be relevant for various physical systems.

A generally accepted idea is that test particle transport in turbulent magnetized plasma is reduced at large Larmor radius. The reason is that the effective motion of the guiding centers is determined by the average of the stochastic potential over cyclotron gyration, which is smaller than local values. It is expected that the diffusion coefficient continuously decreases with the increase of the Larmor radius and that at very large Larmor radii ( $\rho \gg \lambda$ ) it becomes negligible compared with  $D_0$ , the diffusion coefficient obtained at small Larmor radius. The transport of particles with Larmor radii of the order of the correlation length of the turbulence was studied in [7]. It was shown that, in some specified conditions, the reversed effect can appear: the diffusion coefficient of these particles can be sensibly larger than  $D_0$  (see Figure 3). More precisely the diffusion coefficient increases when Larmor radius increases if the time variation of the turbulence is slow.

The reason is that cyclotron motion also determines the build up of correlation of the Lagrangian drift velocity by bringing the particles back in the correlated zone of the stochastic potential. We have shown that the correlation  $L(t)$  shows a series of periodic peaks, which lead to increased diffusion coefficients in slowly varying potentials. Consequently, at given Larmor radius, the transport can be reduced or increased, depending essentially on the value of the Kubo number. The latter characterizes the time variation of the stochastic potential. For a fast time variation of the turbulence corresponding to  $K < 1$ , the diffusion coefficient for Lorentz transport is much smaller than for the drift transport and has a step-like dependence on  $K$ . For a slow time variation of the turbulence with large  $K$  the diffusion coefficient for the particles with Larmor radii of the order of the correlation length is much larger than in the drift approximation.

We have extended the study [7] from the range of Larmor radius comparable to  $\lambda$  to the range of  $\rho$  much larger than  $\lambda$  [8]. Our first aim is to find if there is an effect of the turbulence on the transport of the fast ions produced in the fusion reactions. It is expected that such particles cannot "see" the turbulence since the average of the stochastic potential on the cyclotron gyration is practically zero. We have shown in [8] that this is not always true and that their diffusion coefficient can be much larger than that of slow particles with small Larmor radii,  $D_0$ . We have determined the conditions in which the transport coefficient is comparable or higher than  $D_0$ .

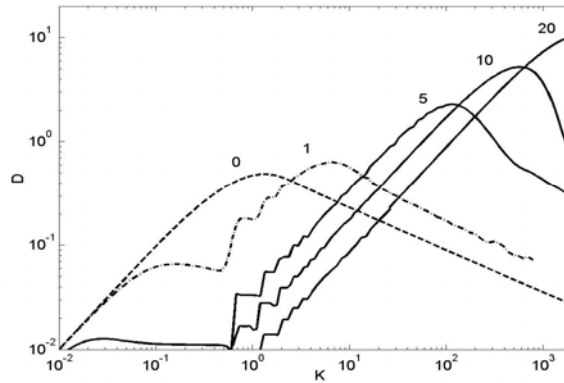


Figure 3: The asymptotic diffusion coefficient as a function of  $K$  for the Lorentz transport at large Larmor radii compared to  $D_0$  (dashed line). The values of  $\rho/\lambda$  label the curves.

We have shown that the quasilinear regime with the diffusion coefficient linear in the Kubo number extends up to large values of  $K$ ,  $K=K_m=2\pi(\rho/\lambda)^2$ . Due to this, transport coefficients much larger than  $D_0$  can be obtained, although the diffusion coefficient is reduced compared to  $D_0$  by a factor  $(2\pi\rho/\lambda)^{-1}$ . The condition is that the stochastic potential has a very slow time variation but it is not static. The maximum of the diffusion coefficient at fixed Larmor radius appears at the "resonance" condition  $K=K_m$ . For  $K>K_m$  the trapping process determines the decay of the diffusion coefficient that is faster than in the drift transport for some interval of  $K$  and, asymptotically follows the same power law as  $D_0$ . In static potential the transport is subdiffusive.

The gyrokinetic approximation was analyzed in this frame of test particle transport and used to understand the above characteristics of the diffusion at large Larmor radii. We have shown that the amplification of the diffusion coefficient is a nonlinear effect determined by the fast gyration motion that averages the stochastic potential leading to a strongly modified shape of the space dependence of the effective correlation.

We have used the decorrelation trajectory method, which is able to take into account the nonlinearity determined by the space dependence of the stochastic potential. This nonlinearity determines a trapping of the guiding center trajectories but with characteristics that are different from those of the drift transport.

The general conclusion of this work is that particles with very large Larmor radii "see" the turbulence if it has slow time variation (large Kubo number).



#### 2.4. Coherent vortical structures and turbulence in tokamak plasma

We have started the analysis of a system of coherent vortices interacting with a background of random drift waves [9], [10], [11]. The method is a development of a previous work<sup>5</sup> and consists of writing the partition function (the generating functional of correlations of the total field) as a product of two factors: one for the dilute gas of vortices with weak mutual interaction and acted upon by the turbulent background, the second for the nonlinearly coupled waves scattered by randomly placed vortices. The generating functional is calculated using path integral methods and an explicit result is possible due to the convergence of the product of eigenvalues of the operators. We obtain a set of results that can be briefly described like a mapping from a certain exponential behavior in the wave number space onto the part of the generating functional that can be associated with it: interaction, nonlinearity, form of the coherent structures, etc.

We have also developed our model for plasma rotation due to soliton self-modulation of the turbulence amplitude<sup>6</sup> and we have prepared a detailed paper [12].

We have shown in a recent paper<sup>4</sup> that trapping or eddying process produced by the ExB drift produces trajectory structures similar to fluid vortices. These results have been developed in [13], [14]. We have extended the analysis based on test trajectories to the study of plasma instabilities. Linear test modes on turbulent plasma for the drift instability in slab geometry with constant magnetic field were analyzed. We have determined a renormalized propagator for these modes, which takes into account the trapping or eddying of the trajectories [15]. The dispersion relation and the growth rate of the test modes are approximately determined as function of the statistical characteristics of the background turbulence. The quasi-coherent component of the ion motion determines the displacement of the unstable range of wave numbers toward smaller values (inverse cascade). It is thus associated with structure formation in turbulent plasmas.

Reviews of our results concerning field theoretical methods in fluids and plasmas [16] and stochastic transport in 2d divergence free velocity fields [17] were presented at the “Albert Einstein Century International Conference”, Paris-UNESCO, France.

### 3. Conclusions

The subjects of the reported work are related with the largely different behaviors can appear in fusion plasmas. Any weak perturbation produces instable modes that develop in turbulent states, which are represented at small amplitudes by random fields (potential, density or currents). However the nonlinear equations that describe plasma at saturation and in absence of dissipations show at large amplitudes coherent structures (soliton-type solution, which are stable and smooth). Between these limits, both these behaviors appear in realistic plasmas, where coherent structures and flows that are intermittently generated in turbulence. We have approached this complex problem from both directions.

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<sup>5</sup> F. Spineanu, M. Vlad, “Spectrum of coherent structures in a turbulent environment”, **Physical Review Letters** **84** (2000) 4854

<sup>6</sup> F. Spineanu, M. Vlad, “Soliton modulation of the turbulence envelope and plasma rotation” **Physical Review Letters** **89** (2002) 185001

Starting from the representation of the turbulence by interacting unstable modes, we have extended the statistical studies by introducing a gas of vortices that interact with the background turbulence [9]. We have determined the spectrum of such states of vortices and turbulence. Another idea was to extend the methods of Dupree based on test trajectories to the strongly nonlinear states characterized by trajectory trapping. The first results obtained for the drift instability [14], [15] show a clear connection between the structures of trajectories produced by trapping<sup>4</sup> and the quasi-coherent structures of potential generated in the evolution of drift turbulence.

Starting from the opposite limit of coherent states, we have developed a field theoretical approach that permitted to determine stationary equations for the asymptotic states reached by the evolution (relaxation) of turbulent states. This approach was confirmed by obtaining analytically the *sinh*-Poisson equation, which is well known from numerical studies to govern the asymptotic states of ideal fluids. An important new result was obtained [1] that consists in determining the equation for the asymptotic states of the drift turbulence described by Hasegawa-Mima equation. A similar approach was used for the study of the density pinch in tokamak produced by the process of vortex merging. The density pinch is related to the fundamental trend to self-organization in two-dimensional plasma [5].

Another series of results concern test particle studies in turbulent plasmas. We have studied the diffusion of particles with large Larmor radii in electrostatic turbulence [7], [8]. A rather unexpected conclusion was obtained, namely that that diffusion coefficients do not decay at very large Larmor radii for any type of turbulence. If the turbulence has slow time variation (large Kubo number) the diffusion coefficient of fast particles can be much larger than in the drift approximation. Very recent numerical simulations of K. Spatschek and G. Zimbardo qualitatively confirm these results. In another study, we have taken into account the inhomogeneity of the confining magnetic field in the ExB transport of test particles and we have shown that an average velocity parallel or anti-parallel with the gradient of the magnetic field is generated by a ratchet type process [6]. This is a fundamental process that is expected to have important effects on instabilities and turbulence evolution.

The objectives assumed in the milestones scheduled for this year have been attained.

#### **4. Results estimated for 2006**

The results obtained this year will be developed in the next period.

The field theoretical approach for determining the asymptotic states of the Hasegawa-Mima equation [1] will be extended to obtain the stationary states in decaying ITG turbulence.

The first results concerning the test particle pinch by a ratchet type process [6] will be developed by study quantitatively the effect of this new pinch mechanism on impurities in tokamak plasmas. We will consider the effects of Large Larmor radius that introduce dependence on mass and charge. The results will be compared with the experimental data obtained on JET.

The results obtained last year on the diffusion induced by Lorentz force in electrostatic turbulence [7], [8] will be extended to the study of electromagnetic turbulence. A realistic model will be developed for the study of alpha particle transport in Alfvén turbulence. This model will include the non-isotropy of the potential correlation, the existence of a dominant mode in the spectrum of the potential and elements of the toroidal geometry. It will be developed by analyzing the numerical results obtained at ENEA-Frascati for the Alfvén modes.

We will also continue our previous work concerning the zonal flows in tokamak plasmas. We have found an exact solution for the zonal flows<sup>7</sup>, which has the characteristics observed in experimental data and in numerical simulation. We will investigate the transient growth (non-normal modes) and the spectrum of the linear perturbations of this solution. The decay of this periodic pattern will be studied numerically and compared with the monopolar structures arising from the Hasegawa-Mima equation.

## **5. Collaborative actions**

### **5.1. Collaboration with ENEA-Frascati**

This collaboration, which started in 2004, has been continued and extended. Two subjects were established for the year 2005:

- Generation and stability of coherent flows in tokamak plasma
- Simulation of alpha particle transport by collective modes

During 2005 three Mobility Secondments were performed: Madalina Vlad and Florin Spineanu at ENEA for 32 days and Silvia Annibaldi at NILPRP in Bucharest for 14 days.

The following results were obtained:

#### **5.1.1 Generation and stability of coherent flows in tokamak plasma**

This subject is based on a previous work of our group<sup>7</sup> where an exact solution of the nonlinear equation describing the late stage of the ion dynamics (Flierl-Petviashvili) has been identified. There is an effective synergy with the experience of the Italian group who has developed analytical and numerical techniques for describing zonal flows. The collaboration has to provide an effective instrument to quantitatively explain the formation of the poloidally symmetric flow from the initially radial eddies of the ion motion, via the Reynolds stress. This would represent an alternative scenario for the formation of Internal Transport Barrier. The collaboration has requested many interactions since the approaches of the two groups were essentially different: the group from Frascati together with scientists from Princeton and Irvine, USA have developed a model for zonal flows based on a different scenario, known as the four-wave model.

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<sup>7</sup> F. Spineanu, M. Vlad, K. Itoh, H. Sanuki, S.-I. Itoh, "Pole dynamics for the Flierl-Petviashvili equation and zonal flows", **Physical Review Letters** **93** (2004) 025001.

The present situation in the development of this idea in our collaboration can be summarized as follows: the tilting instability can be invoked to explain the reorientation along the poloidal direction of the ITG eddies. It is still necessary to provide a detailed model for the coalescence of eddies. We have decided with our collaborators to look to the Eckhaus instability and eventually to use Liapunov functional and to reduce the problem to a barrier-type, since it is expected that the sheared flow has a lower energy compared with the tilted separated eddies and that these two states are separated by a local maximum of the energy.

The meeting of November 2005 has permitted the extension of the subjects of common interest: a possible explanation of recent experimental observation (at ENEA-Frascati but also at Garching, Jaeri, Doublet IIID) that has shown the existence of a magnetic oscillation around the stationary island resulted from the tearing instability, with a frequency of about 50 kHz. This oscillation may have as origin the geodesic acoustic mode, an oscillatory sheared flow (of a nature very similar to what has been observed by Japan scientists and attributed to zonal flows). The aim of our common work is to find an explanation for this phenomenon and to correctly associate it with a previously known process. We will examine the possible excitation of the Alfvén wave close to the tearing mode and for this it is necessary to provide an explanation of the enhanced resistivity in the current sheet part of the tearing mode. We have analyzed a new mechanism for non-collisional enhanced resistivity in the X-point region of the tearing mode. As the X-point evolves to a current sheet a stochastic magnetic field is generated. The stochastic structure can be associated with a linking of the magnetic flux tube and quantitative estimation of the topological degree of entanglement can be obtained. The topological linking can be formally converted into a spatial distribution of the local curvature of the magnetic flux tubes. The space-fluctuation of the local curvature produces a space-fluctuation of the difference in the relative drifts of the ions and electrons, thus a transversal current on the local magnetic line. This current combines with a small but not vanishing resistivity, thus altering the rate of momentum transfer and obtaining a physical new process leading to enhanced resistivity. The enhanced resistivity is one of the most complex aspects of the tearing mode evolution, not understood yet. We hope this will be a contribution to clarify this problem.

### **5.1.2 Simulation of alpha particle transport by collective modes**

The main objective is to connect the general studies of test particle transport performed in our group in Bucharest [7], [8] with the numerical simulation of Alfvén modes, which is one of the topics studied in theory group at Frascati. We have determined the diffusion coefficient of alpha particles in an electrostatic turbulence with given Eulerian correlation (spectrum) starting from Lorentz force [7] and from the gyrokinetic approximation [8]. The decorrelation trajectory method developed for the study of transport induced by ExB drift was extended to the equation of motion of second order with Lorentz force. Simple Eulerian correlations were considered in these studies, corresponding to isotropic turbulence without a dominant mode, in slab geometry. In another work [7] we have shown that due to the trapping or eddying specific to the ExB drift, particle trajectories have a special statistics with memory effects and quasi-coherent behaviour of a part of trajectories that form structures similar with fluid vortices. Recently, we have shown (in a particular study for the drift instabilities) that there is a strong connection between these trajectory structures and the potential structures that appear

in the evolution of the turbulence [15]. This was obtained by developing a Lagrangian method for the test modes in turbulent plasma, which takes into account trajectory trapping. The statistical quantities that appear in this method could be numerically obtained from a PIC code. We think that such a combined study (analytical and numerical) could provide a better understanding of the nonlinear processes appearing in turbulence. The general aim of the collaboration between Bucharest and Frascati theory groups is to develop the statistical analysis of alpha particles for the case of Alfvén instabilities using the numerical code of Frascati.

We have decided that the first step is to construct a model for test particle studies that is in agreement with the characteristics of the turbulence generated by Alfvén modes. This means that in this stage we model the Eulerian correlations by analytical functions, which are obtained by analysing the numerical simulations. This realistic model is obtained from the basic one considered in our previous work by adding the following factors: the magnetic potential and the stochastic component of the magnetic field in the trajectory equations, the non-isotropy of the potential correlation, the existence of a dominant mode in the spectrum of the potential and elements of the toroidal geometry (periodicity in the poloidal and toroidal directions, resonance condition).

We have obtained general conclusions about the effects of the non-isotropy of the electrostatic turbulence and of the presence of a dominant mode. More precisely we have considered different correlation lengths on x and y directions (radial and poloidal) and an attenuated oscillatory behaviour of the Eulerian correlation with wave numbers that can be along x and/or y-axis or along arbitrary direction (tilting of potential cells).

## 5.2. Collaboration with CEA-Cadarache and Université de Provence

The collaboration with CEA-Cadarache started in 1992 and this year it was extended by including the “Dynamics of Complex Systems Group” from Université de Provence. Our Plasma Theory Group of the National Institute of Laser, Plasmas and Radiation Physics (Association EURATOM-MEdC) has several common research topics with these two groups. The collaboration subject for 2005 is decorrelation trajectory method and nonlinear turbulent effects.

During 2005 one Mobility Secondment was performed: Madalina Vlad at CEA-Cadarache and Université de Provence for 45 days.

We have analyzed the possibility that average velocities (pinches) appear for particles moving in turbulent plasmas. The main application concern impurity accumulation in the central region of the plasma, which is a very important issue for the development of fusion reactors.

We have found an alternative mechanism, which shows that an average velocity is produced in turbulent plasmas through a ratchet-type process due to the space variation of the confining magnetic field [6]. The velocity and the diffusion coefficient were obtained for space dependent magnetic fields using our semi-analytical approach, the decorrelation trajectory method.

We have shown that the inhomogeneity of the confining magnetic field determines a directed transport (an average velocity), although the average ExB drift is zero. We have shown that an interesting nonlinear effect appears in the presence of trajectory trapping: the average velocity changes its direction.

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